Adiabatic Pulse Design

The following pages contain excerpts from the lecture notes for "RF Pulse Design for Magnetic Resonance Imaging," a course taught at Stanford University by John M. Pauly.

ADIABATIC PULSES

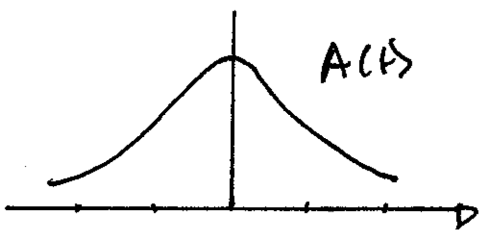
FREQUENCY MODULATED PULSES OF THE FORM

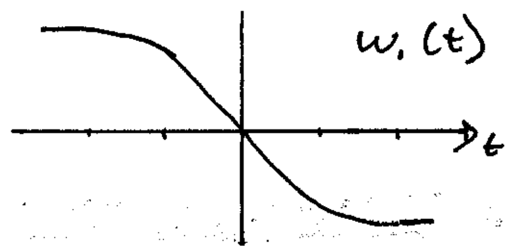
$$B_1(t) = A(t)\, e^{-i\omega_1(t)\,t}$$

WHERE

$A(t)$ - ENVELOPE

$\omega_1(t)$ - FREQUENCY SWEEP





MANY DIFFERENT OPTIONS. MOST FAMOUS

$$A(t) = A_0 \operatorname{sech}(\beta t)$$

$$\omega_1(t) = -\mu\beta \tanh(\beta t)$$

μ, β PARAMETERS. <u>SILVER-HOULT</u> PULSE, OR <u>HYPERBOLIC SECANT</u>.

ONE OF THE FEW ANALYTIC SOLUTIONS
TO THE BLOCH EQUATION

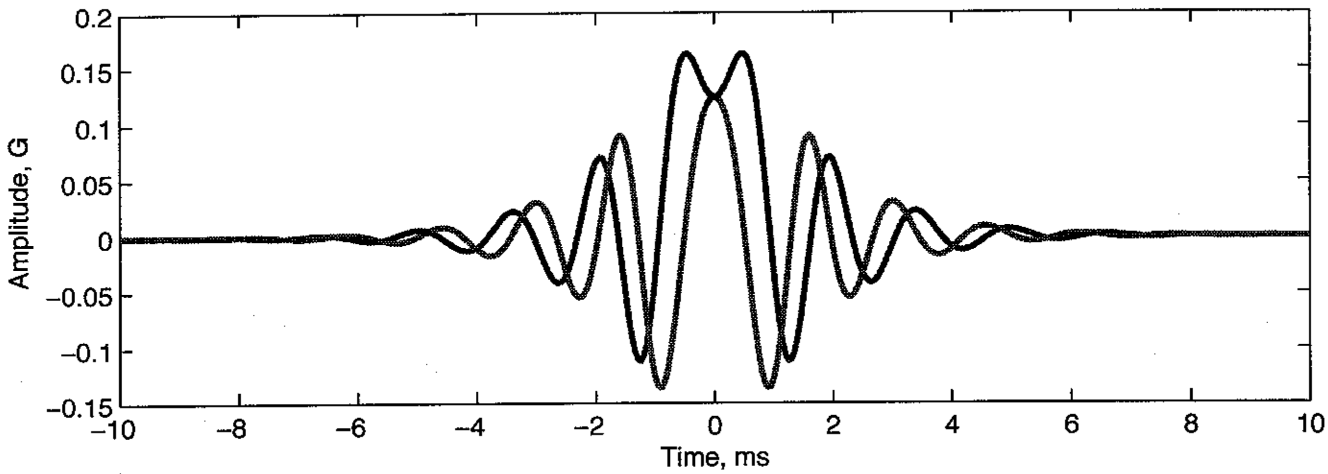
FIRST OF A FAMILY OF SOLUTIONS CALLED
SOLITONS.

<u>REMARKABLE CHARACTERISTIC</u>

A HYPERBOLIC SECANT PULSE PERFORMS
A FREQUENCY (SLICE) SELECTIVE INVERSION
FOR <u>ANY</u> RF AMPLITUDE ABOVE A
THRESHOLD.

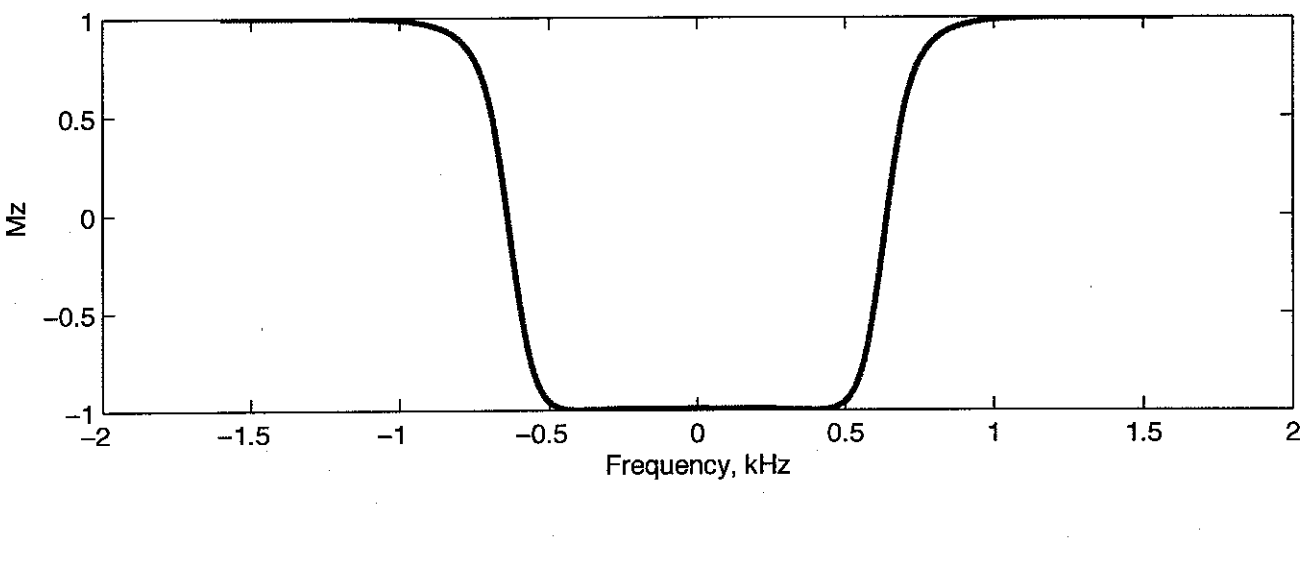
Hyperbolic Secant

RF Pulse

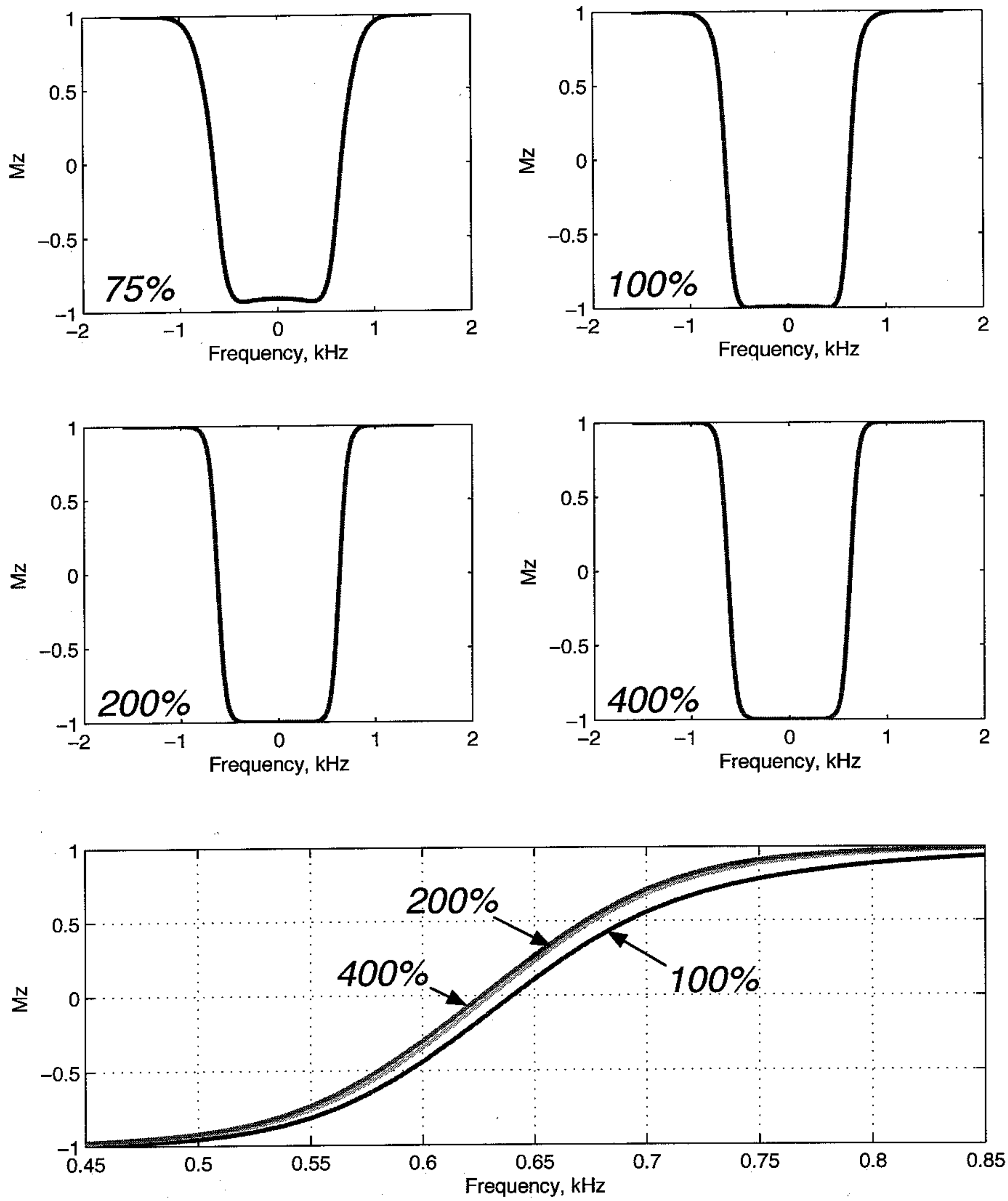


$$B_1(t) = A_0 sech(\beta t) e^{-i\mu\beta \tanh(\beta t) t}$$
$$\beta = 800\ rad/s; \quad \mu = 4.9$$

Inversion Profile



Inversion Profiles



ROTATING REFERENCE FRAMES

BLOCH EQUATION IN LABORATORY FRAME

$$\left(\frac{d\underline{m}}{dt}\right) = -\gamma \underline{B} \times \underline{m}$$

WHERE

$$\underline{B} = B_1 \cos\omega_0 t\, \hat{i}_0 + B_1 \sin\omega_0 t\, \hat{j}_0 + B_0 \hat{k}_0$$

AND $\hat{i}_0, \hat{j}_0, \hat{k}_0$ ARE FIXED UNIT VECTORS IN LAB FRAME, AND $B_1 = B_{1,x}$ FOR CONVENIENCE

WE WANT TO DESCRIBE THE MAGNETIZATION IN A FRAME ROTATING AT ω_0

$$\hat{i} = \hat{i}_0 \cos\omega_0 t + \hat{j}_0 \sin\omega_0 t$$

$$\hat{j} = -\hat{i}_0 \sin\omega_0 t + \hat{j}_0 \cos\omega_0 t$$

$$\hat{k} = \hat{k}_0$$

THEN

$$\underline{B} = B_1 \hat{i} + B_0 \hat{k}$$

$$\underline{m} = m_x \hat{i} + m_y \hat{j} + m_z \hat{k}$$

BOTH (m_x, m_y, m_z) AND $\hat{i}, \hat{j}, \hat{k}$ TIME VARYING IN LAB FRAME.

DIFFERENTIATING

$$\left(\frac{d}{dt}\underline{m}\right)_{LAB} = \frac{\partial m_x}{\partial t}\hat{i} + \frac{\partial m_y}{\partial t}\hat{j} + \frac{\partial m_z}{\partial t}\hat{k}$$

$$+ m_x\frac{\partial \hat{i}}{\partial t} + m_y\frac{\partial \hat{j}}{\partial t} + m_z\frac{\partial \hat{k}}{\partial t}$$

$$= \left(\frac{\partial \underline{m}}{\partial t}\right)_{ROT} + m_x\frac{\partial \hat{i}}{\partial t} + m_y\frac{\partial \hat{j}}{\partial t} + m_z\frac{\partial \hat{k}}{\partial t}$$

$\hat{i}, \hat{j}$ ARE ROTATING UNIT VECTORS WHICH WE CAN REPRESENT AS A CROSS PRODUCT WITH

$$\underline{\omega}_0 = \omega_0\hat{\underline{k}}$$

SO

$$\frac{\partial \hat{i}}{\partial t} = \underline{\omega}_0 \times \hat{i}\ ; \quad \frac{\partial \hat{j}}{\partial t} = \underline{\omega}_0 \times \hat{j}\ ; \quad \frac{\partial \hat{k}}{\partial t} = \underline{\omega}_0 \times \hat{k}$$

SUBSTITUTING

$$\left(\frac{d}{dt}\underline{m}\right)_{LAB} = \left(\frac{\partial \underline{m}}{\partial t}\right)_{ROT} + \underline{\omega}_0 \times \left(m_x\hat{i} + m_y\hat{j} + m_z\hat{k}\right)$$

$$= \left(\frac{d m}{d t}\right)_{ROT} + \underline{\omega}_0 \times \underline{m}$$

IN THE ROTATING FRAME

$$\left(\frac{\partial \underline{m}}{\partial t}\right)_{ROT} = \frac{d}{dt}\underline{m} - \underline{\omega_0} \times \underline{m}$$

$$= -\gamma \underline{B} \times \underline{m} - \underline{\omega_0} \times \underline{m}$$

$$= \left(-\gamma \underline{B} - \underline{\omega_0}\right) \times \underline{m}$$

$$= -\gamma \underbrace{\left(\underline{B} + \frac{\underline{\omega_0}}{\gamma}\right)}_{\underline{B}_{eff}} \times \underline{m}$$

AT THE LARMOR FREQUENCY

$$\underline{\omega_0} = -\gamma B_0 \hat{k}$$

AND

$$\underline{B} = B_1 \cos \omega_0 t \, \hat{\imath}_0 + B_1 \sin \omega_0 t \, \hat{\jmath}_0 + B_0 \hat{k}_0$$

$$= B_1 \hat{\imath} + B_0 \hat{k}$$

SO

$$\underline{B}_{eff} = B_1 \hat{\imath} + \cancel{B_0 \hat{k}} + \frac{-\gamma \cancel{B_0}}{\cancel{\gamma}} \hat{k}$$

$$= B_1 \hat{\imath}$$

THE RESULT IS THAT THE STATIC B_0 FIELD HAS BEEN "DELETED" IN ROTATING FRAME!

KEY IDEA

CHANGING REFERENCE FRAMES BY A FREQUENCY ω CORRESPONDS TO ADDING A Z-FIELD OF $\frac{\omega}{\gamma}$.

IN PARTICULAR IF I HAVE AN RF PULSE

$$B_1(t) = A(t)\, e^{i\omega_1(t)\, t}$$

AT THE LARMOR FREQUENCY, THEN

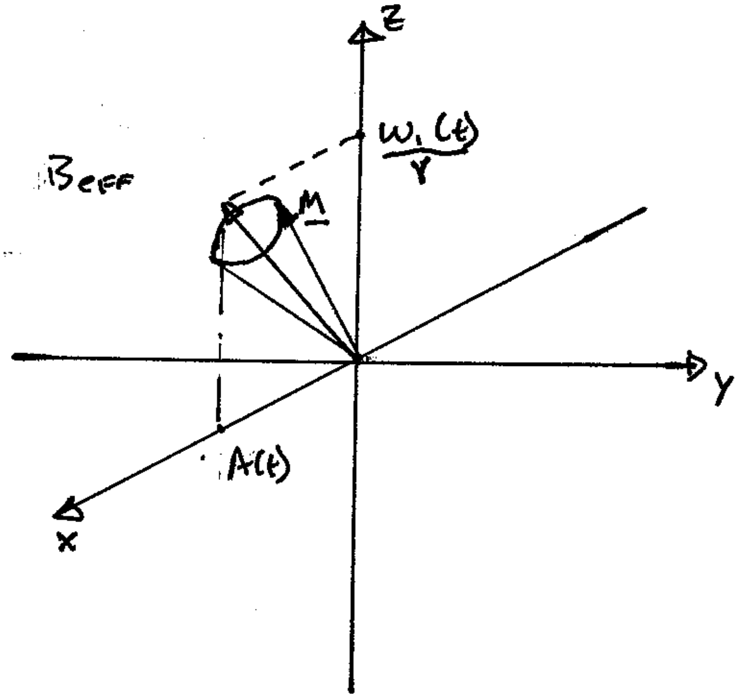
$$(\underline{B})_{\omega_0} = A(t)\cos\omega_1(t)\,\hat{i}_{\omega_0} + A(t)\sin\omega_1(t)\,\hat{j}_{\omega_0} + 0\,\hat{k}$$

IF WE CHANGE TO THE ROTATING FRAME AT $\omega_0 + \omega_1(t)$

$$(\underline{B})_{\omega_0+\omega_1(t)} = A(t)\,\hat{i}_{\omega_0+\omega_1(t)} + \frac{\omega_1(t)}{\gamma}\hat{k}$$

$$= \left(A(t),\ 0,\ \frac{\omega_1(t)}{\gamma}\right)$$

MAGNETIZATION PLOT



MAGNETIZATION PRECESSES ABOUT $\underline{B}_{eff}$

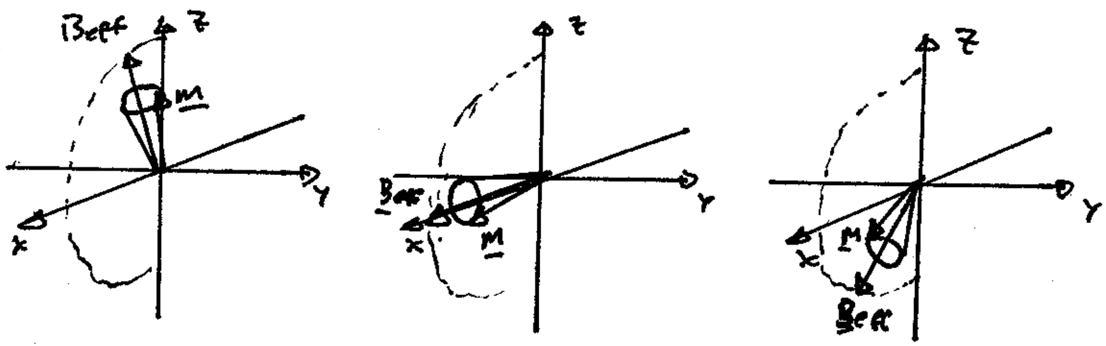
$$\underline{B}_{eff} = \left(A(t),\ 0,\ \frac{\omega_1(t)}{\gamma}\right)$$

$$|B_{eff}| = \sqrt{A^2(t) + \left(\frac{\omega_1(t)}{\gamma}\right)^2}$$

$$\angle B_{eff} = \psi = \tan^{-1}\left(\frac{A(t)}{\omega_1(t)/\gamma}\right)$$

BASIC IDEA OF ADIABATIC PULSES

GIVEN INITIAL $\underline{M}$ ALONG +Z AXIS, SWEEP $\underline{B}_{eff}$ FROM +Z TO −Z SLOWLY ENOUGH THAT $\underline{M}$ FOLLOWS $\underline{B}_{eff}$



$\underline{M}$ SHOULD PRECESS FASTER THAN THE ANGLE OF $\underline{B}_{eff}$ CHANGES, $\frac{d\psi}{dt}$

$$\left|\frac{d\psi}{dt}\right| \ll \gamma\,|B_{eff}|$$

ADIABATIC CONDITION. DEFINE

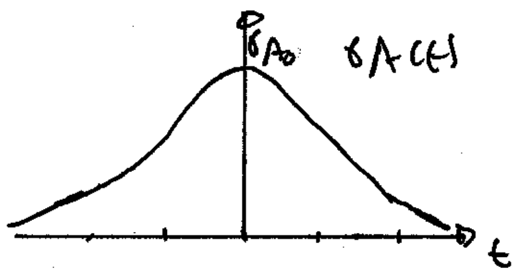
$$\eta = \frac{\gamma\,|B_{eff}|}{\left|\frac{d\psi}{dt}\right|}$$

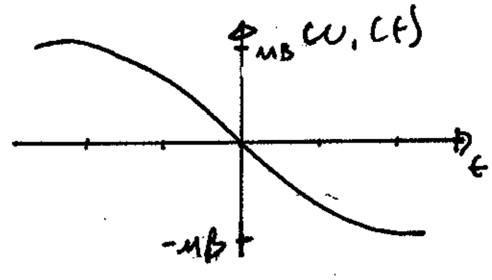
AS ADIABATIC FACTOR, LARGER THAN 1.

SWEEP DIAGRAMS

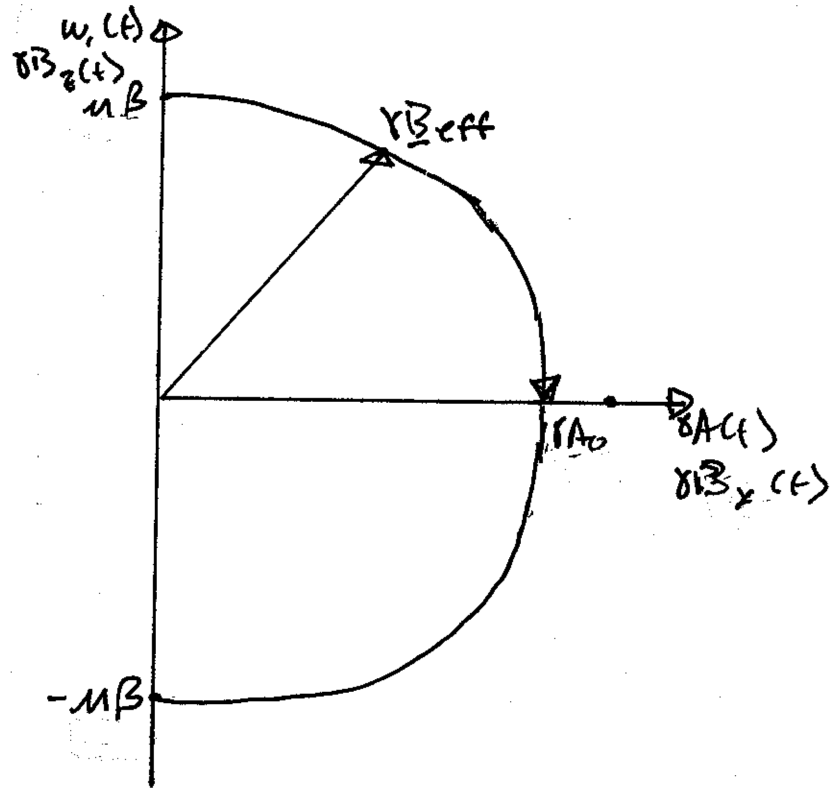
VERY USEFULL TOOL FOR UNDERSTANDING ADIABATIC PULSES. DUE TO CONOLLY.

PLOT $\gamma A(t)$ VS $\omega_1(t)$

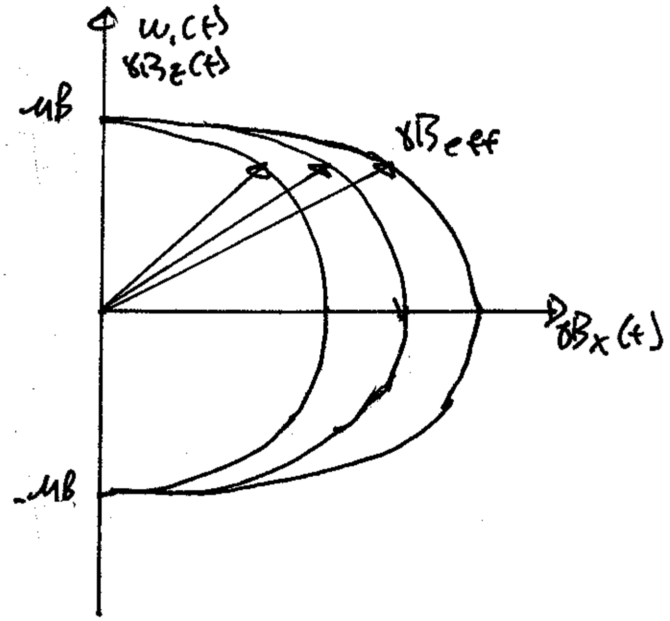




$\gamma A(t) = \gamma A_0 \operatorname{sech}(\beta t)$ $\qquad$ $\gamma\omega_1(t) = -\mu\beta \tanh(\beta t)$



ADIABATIC PULSE ARE INSENSITIVE TO B_1 SCALING, PROVIDED ADIABATIC CONDITION IS MET



ALL PRODUCE INVERSIONS PROVIDED

$$\frac{d\varphi}{dt} \ll \gamma |B_{eff}|$$

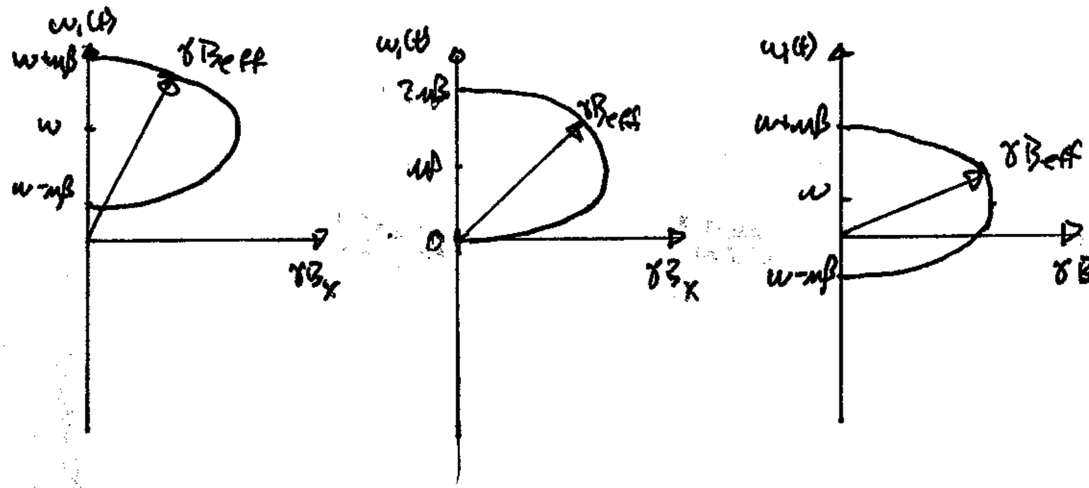
AS B_1 INCREASES, CAN LOOSE ADIABATICITY AT THE BEGINNING AND END.

ADIABATICITY IMPROVES IN THE MIDDLE.

ADIABATIC PULSES ARE SELECTIVE

FREQUENCY OR SLICE SELECTIVE

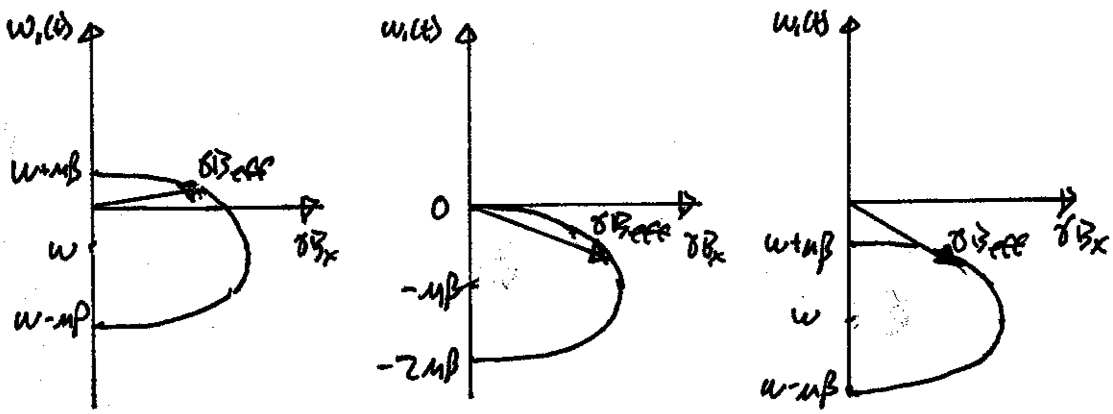
DETERMINED BY SWEEP BANDWIDTH



$\omega > \mu\beta$ — NOT INVERTED

$\omega = \mu\beta$ — TRANSITION BAND (NOT ADIABATIC)

$-\mu\beta < \omega < \mu\beta$ — INVERTED



$-\mu\beta < \omega < \mu\beta$ — INVERTED

$\omega = -\mu\beta$ — TRANSITION BAND (NOT ADIABATIC)

$\omega < -\mu\beta$ — NOT INVERTED

<u>ANTI-PARALLEL</u>

FOR $\omega > \mu\beta$ OR $\omega < -\mu\beta$, $\underline{M}$ RETURNED TO $+z$ AXIS

FOR $-\mu\beta < \omega < \mu\beta$ $\underline{M}$ IS INVERTED

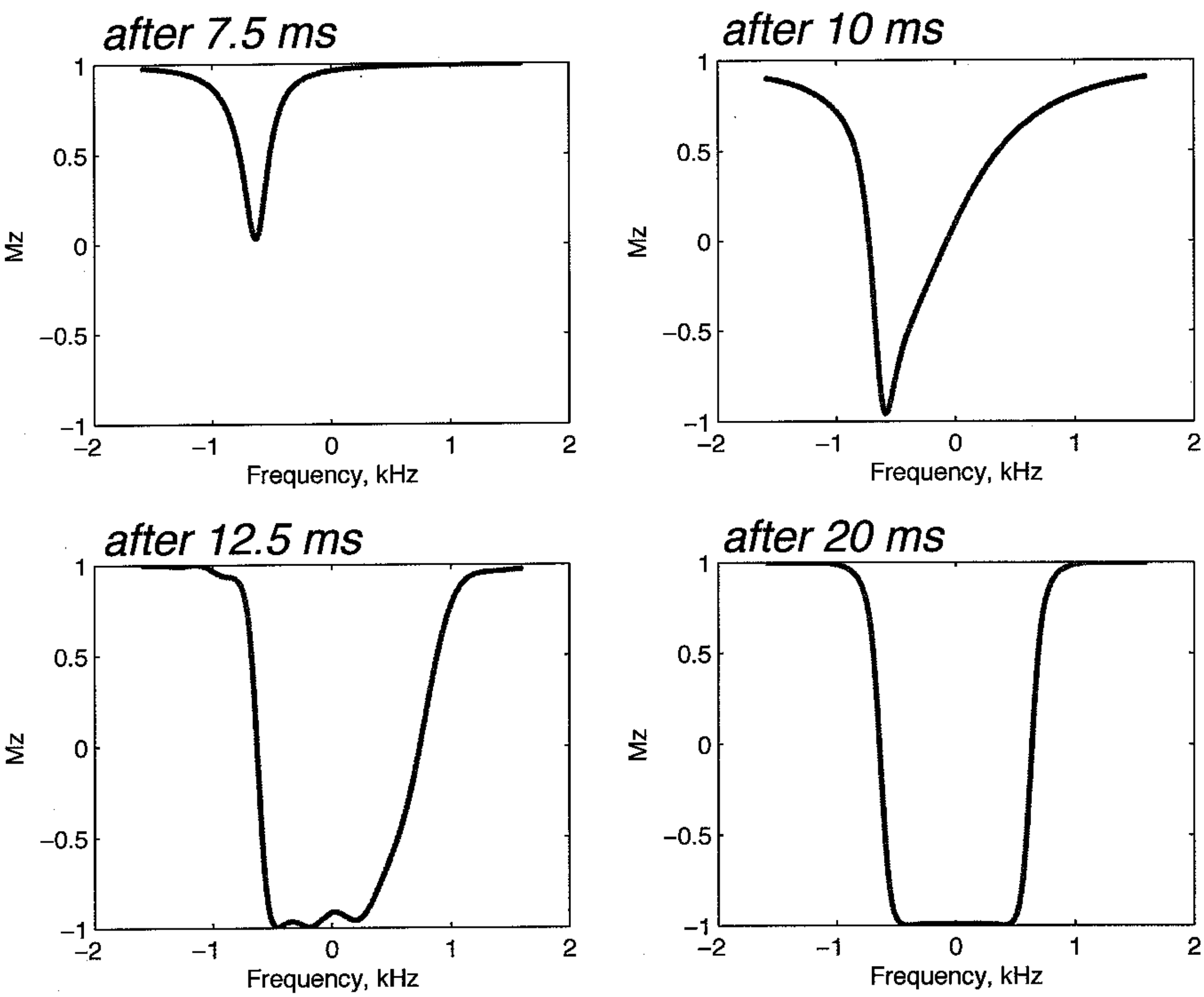
BANDWIDTH IS

$$\Delta f = \frac{2\mu\beta}{2\pi}$$

$$= \underline{\frac{\mu\beta}{\pi}} \quad \text{HZ}$$

BAND EDGE CLOSEST TO BEGINNING OF FREQUENCY SWEEP DEFINED FIRST, OTHER BAND EDGE LAST

Inversion Sequence



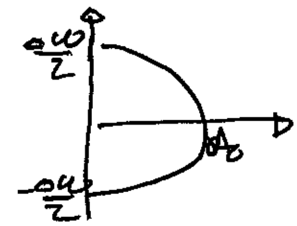
Comments

1) Many envelope/modulation functions work

Ex:

$$A(t) = A_0 \sin(\omega_2(t)t)$$

$$\omega_1(t) = \frac{\Delta\omega}{2}\cos(\omega_2(t)t)$$



where Δf is the bandwidth.

2) If a range of adiabaticity is required, optimization can help reduce pulse length.

3) Hyperbolic sech's need to be truncated, can significantly effect performance.

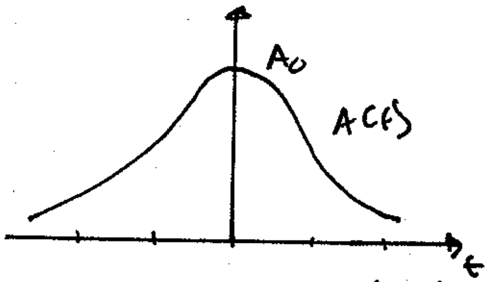
ADIABATIC INVERSIONS: REVIEW

FREQUENCY MODULATED PULSE

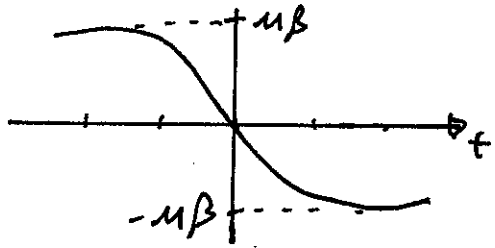
$$B_1(t) = A(t)\, e^{-i\omega_1(t) t}$$

IN ROTATING FRAME AT LARMOR FREQUENCY

TYPICAL WAVEFORMS:



$$A(t) = A_0 \operatorname{sech}(\beta t)$$



$$\omega_1(t) = -\mu\beta \tanh \beta t$$

MANY OTHER POSSIBILITIES

IN THE ROTATING FRAME AT ω_0

$$\underline{B}(t) = \left(A(t) \cos \omega_1(t) t,\ A(t) \sin \omega_1(t) t,\ 0 \right)$$

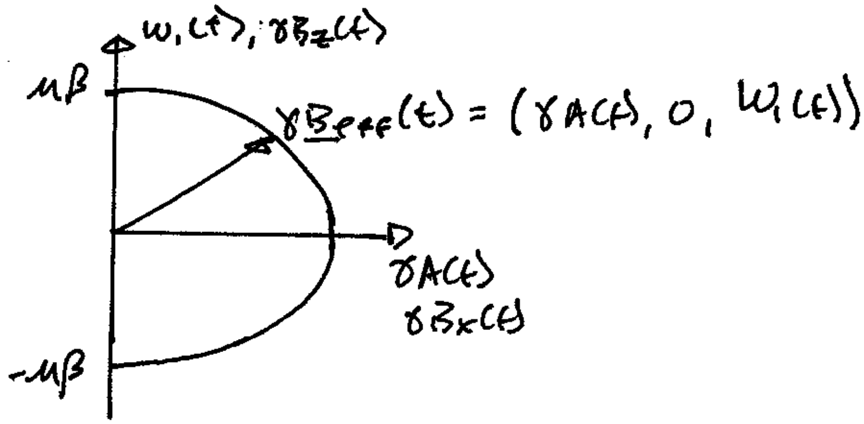
IN THE ROTATING FRAME AT $\omega_0 + \omega_1(t)$

$$\underline{B}_{eff}(t) = \left(A(t),\ 0,\ \frac{\omega_1(t)}{\gamma} \right)$$

FREQUENCY MODULATION OF $B_1(t)$ CORRESPONDS TO Z-FIELD IN ROTATING FRAME AT MODULATION FREQUENCY.

SWEEP DIAGRAM

PLOT OF $\underline{B}_{eff}(t)$

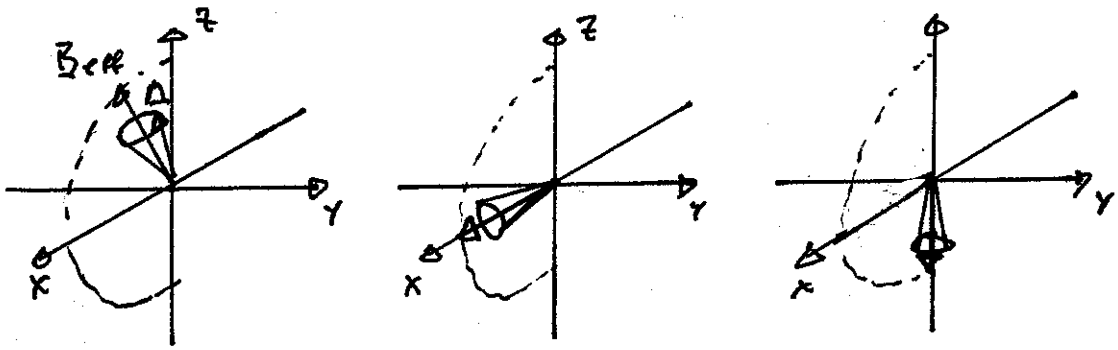


$\underline{B}_{eff}$ SWEEP FROM $+z$ TO $-z$

$\underline{M}$ FOLLOWS, PROVIDED $\psi = \angle B_{eff}(t)$ DOESN'T CHANGE TOO FAST COMPARED TO $|B_{eff}(t)|$

$$\left| \frac{d\psi}{dt} \right| \ll \gamma \left| B_{eff}(t) \right|$$

ADIABATIC CONDITION.



$\underline{M}$ TRACKS $\underline{B}_{eff}(t)$

OTHER ADIABATIC PULSES

EXCITATION

SPIN-ECHO

ROTATION BY ARBITRARY ANGLE

EXCITATION

WHY NOT SCALE ADIABATIC INVERSION TO $\pi/2$?

TWO ANSWERS:

1) BELOW ADIABATIC THRESHOLD (i.e. π PULSE) AN "ADIABATIC" PULSE IS THE SAME AS ANY OTHER PULSE, WITH

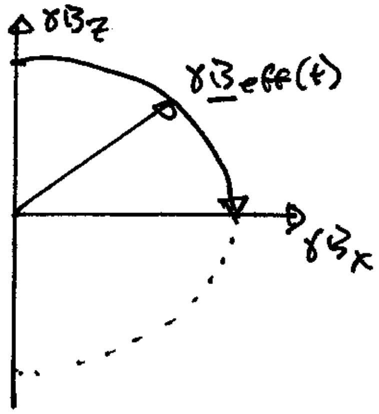
$$\Theta = \int_0^T \gamma B_1(t)\,dt$$

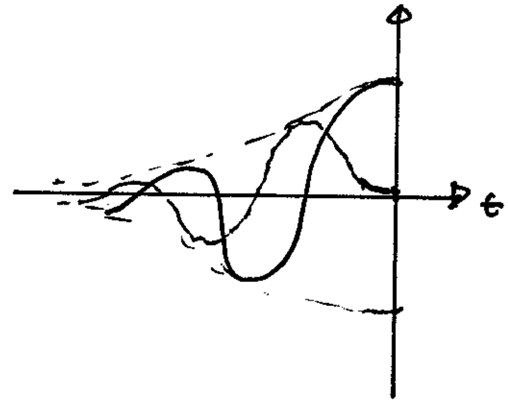
WHICH IS NOT B_1 INSENSITIVE

2) ADIABATIC PULSES ARE GENERALLY APPROXIMATELY QUADRATIC PHASE, DON'T REFOCUS.

SIMPLE ADIABATIC EXCITATION

ADIABATIC HALF PASSAGE: HALF OF AN INVERSION SWEEP.

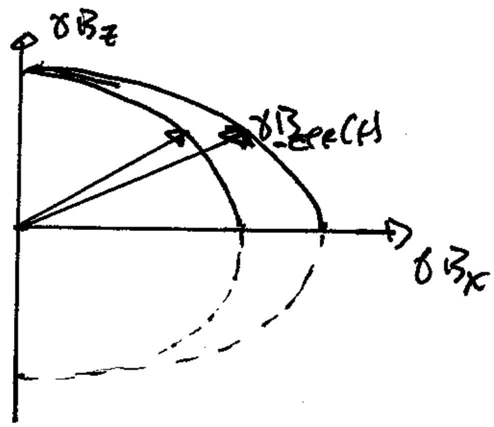




FIRST HALF OF HYPERBOLIC SECANT

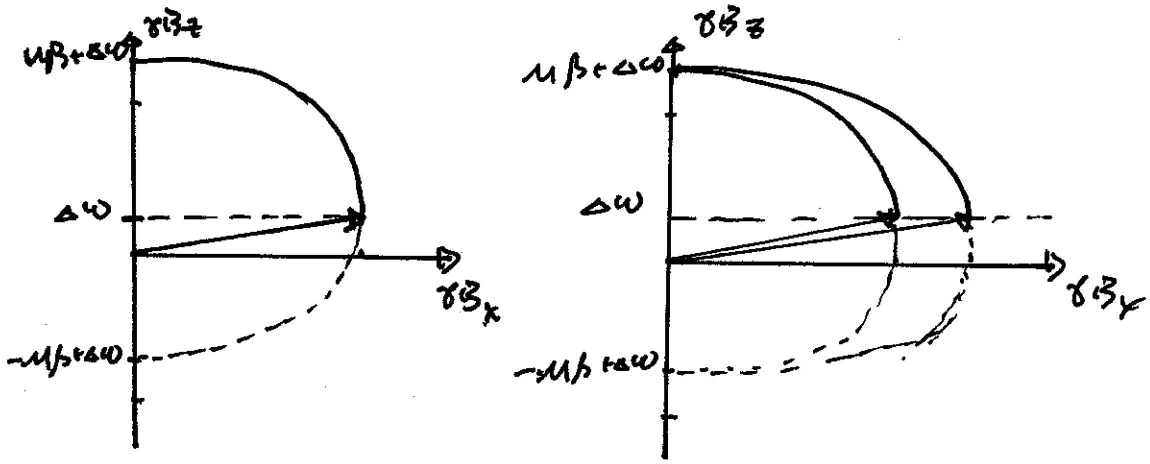
$$\left.\begin{aligned} A(t) &= A_0 \operatorname{sech}(\beta t) \\ \omega_1(t) &= -\mu\beta \tanh(\beta t) \end{aligned}\right\} -\infty < t < 0$$

B_1 INSENSITIVE, ABOVE ADIABATIC THRESHOLD



$\underline{M}$ LEFT ALONG +X AXIS

SENSITIVE TO OFF-RESONANCE



OFF-RESONANCE SHIFT OF $\Delta\omega$ LEAVES $\underline{B}_{eff}$ AND $\underline{M}$ AT AN ANGLE

$$\phi = \tan^{-1}\left(\frac{\Delta\omega}{\gamma A_0}\right)$$

ABOVE THE TRANSVERSE PLANE

ϕ DECREASES AS B_1 INCREASES.

WORKS REASONABLY WELL FOR CREATING M_{xy} INSENSITIVE TO ϕ

WORKS VERY POORLY FOR ELIMINATING M_z, (i.e SATURATION PULSES)

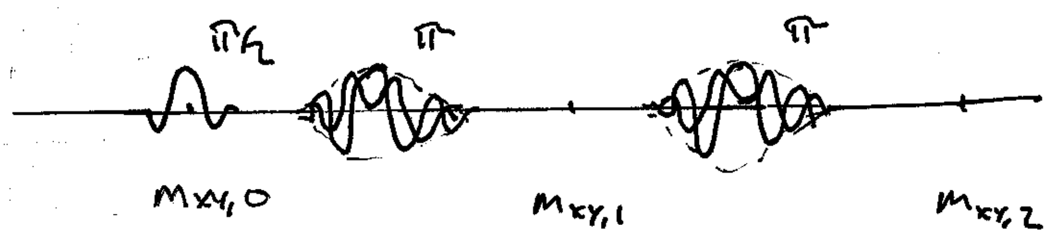
ADIABATIC REFOCUSING PULSES

WHY NOT USE AN ADIABATIC INVERSION?

1) ACCURATELY RETURNS TRANSVERSE MAGNETIZATION TO TRANSVERSE PLANE

2) <u>ADDS NON-LINEAR PHASE</u>

<u>SIMPLE SOLUTION</u>: USE A PAIR OF ADIABATIC INVERSIONS



THE ADIABATIC π PRODUCES A ROTATION DESCRIBED BY (α_π, β_π)

$$M_{xy,1} = (M_{xy,0})^* \beta_\pi^2$$

$$\begin{aligned} M_{xy,2} &= (M_{xy,1})^* \beta_\pi^2 \\ &= \left((M_{xy,0})^* \beta_\pi^2 \right)^* \beta_\pi^2 \\ &= M_{xy,0} \, |\beta_\pi|^4 \end{aligned}$$

AS LONG AS WE USE TWO IDENTICAL REFOCUSING PULSES, PHASE PROFILES CANCEL.

CHANGES IN B_1 AMPLITUDE WILL CHANGE BY PHASE, BUT DOESN'T MATTER

DISADVANTAGE:

MUST FORM FIRST ECHO

LONGER DELAY UNTIL FIRST USEFUL ECHO, WHICH IS THE SECOND

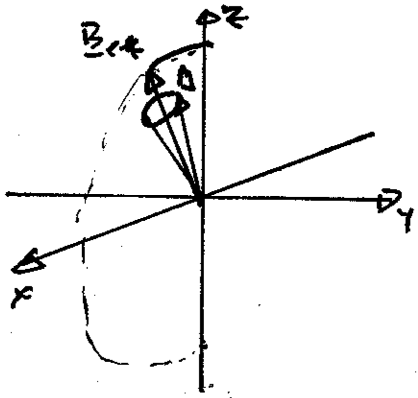
ONLY EVEN ECHOES USABLE

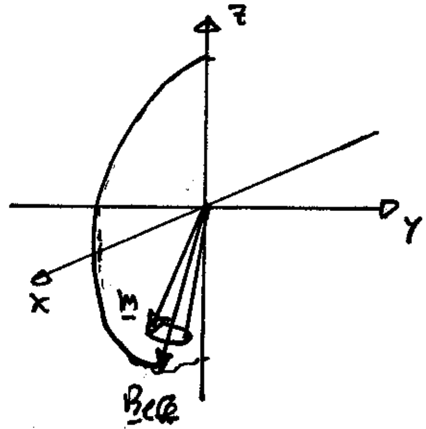
IDEALLY WE WANT A SINGLE PULSE.

BASIC TOOLS FOR ADIABATIC PULSE DESIGN

FOR ADIABATIC SWEEPS:

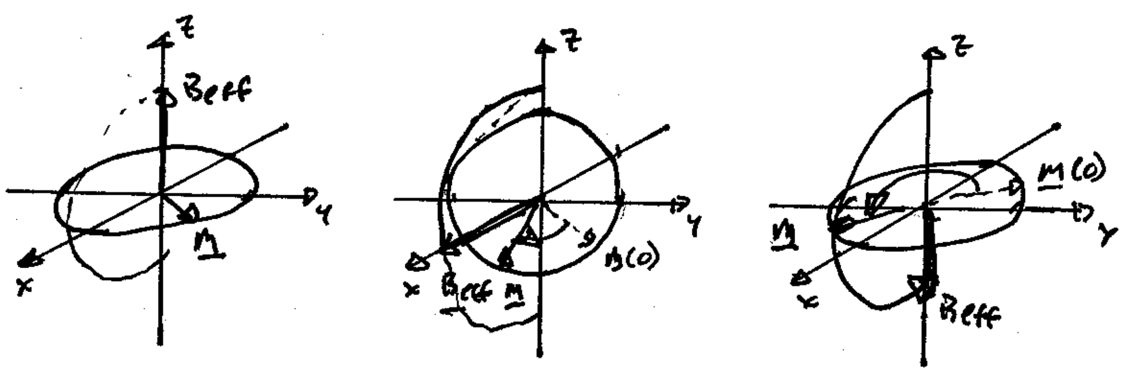
PARALLEL COMPONENT STAYS PARALLEL





THIS IS WHAT WE USE FOR ADIABATIC INVERSION PULSES

PERPENDICULAR COMPONENT STAYS PERPENDICULAR



FOR INITIAL TRANSVERSE MAGNETIZATION

1) INVERTED

2) ADDITIONAL PHASE DUE TO PRECESSION

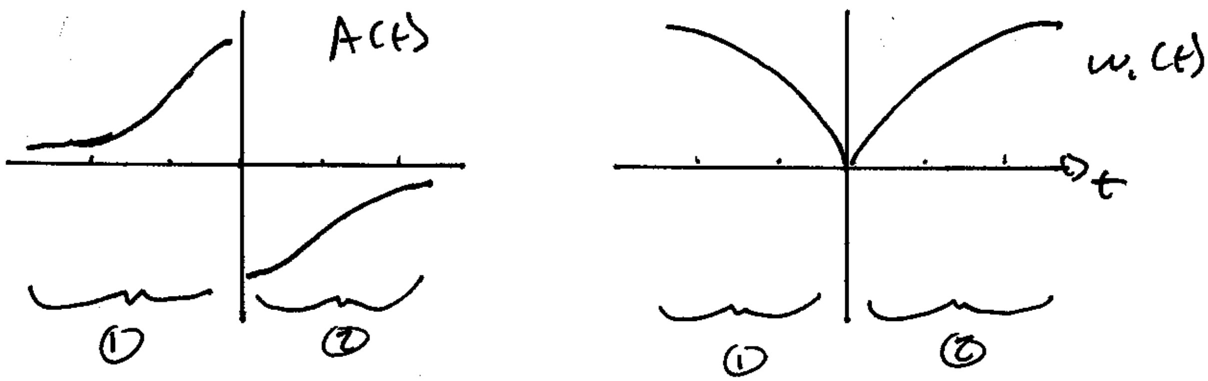
$$\theta_{eff} = \int_0^T \gamma |\underline{B}_{eff}(t)| \, dt$$

REFOCUSING PULSES COMBINE PARTIAL SWEEPS TO DO THE INVERSION WHILE CANCELING OUT THE PRECESSION PHASE SHIFTS

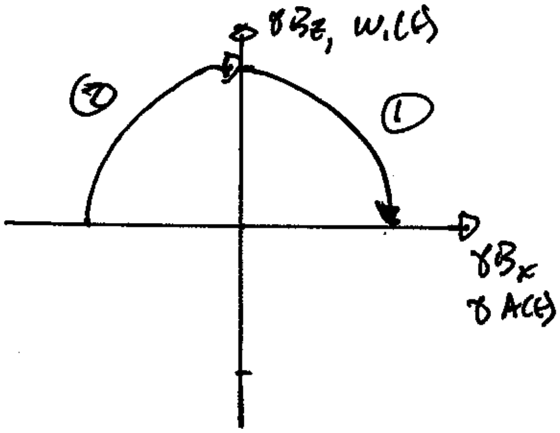
SIMPLE ADIABATIC REFOCUSING PULSE

B_1 INSENSITIVE REFOCUSING PULSE - 1 (BIREF-1)

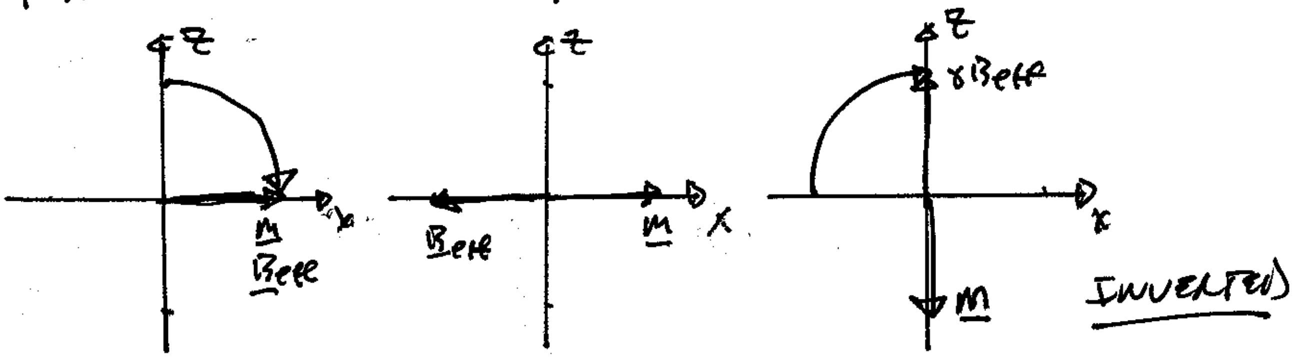
ENVELOPE / MODULATION WAVEFORMS



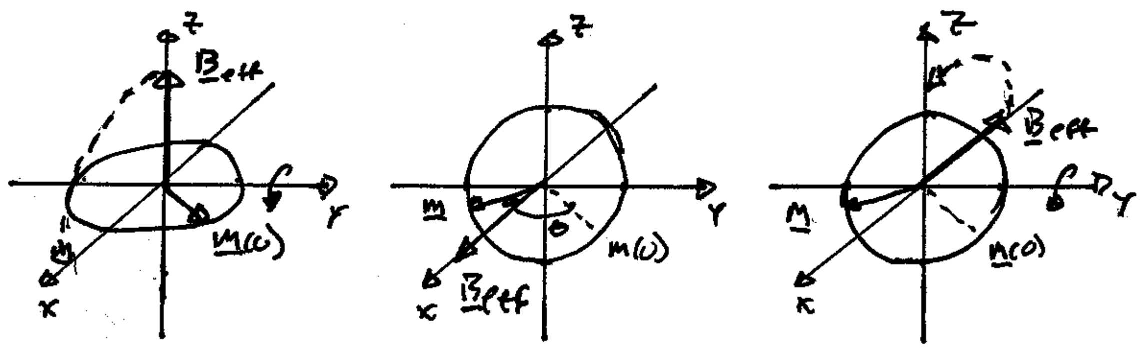
SWEEP DIAGRAM

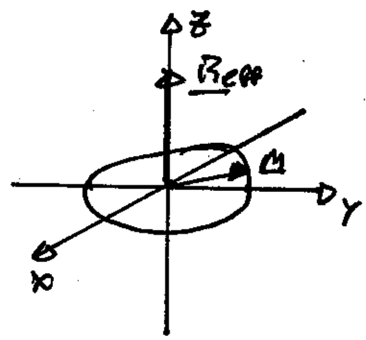


PARALLEL COMPONENT, INITIALLY ALONG +Z



PERPENDICULAR COMPONENT, INITIALLY TRANSVERSE





$\underline{M}$ HAS BEEN REFLECTED ABOUT +Y AXIS!

RESULT IS A SPIN-ECHO PULSE.

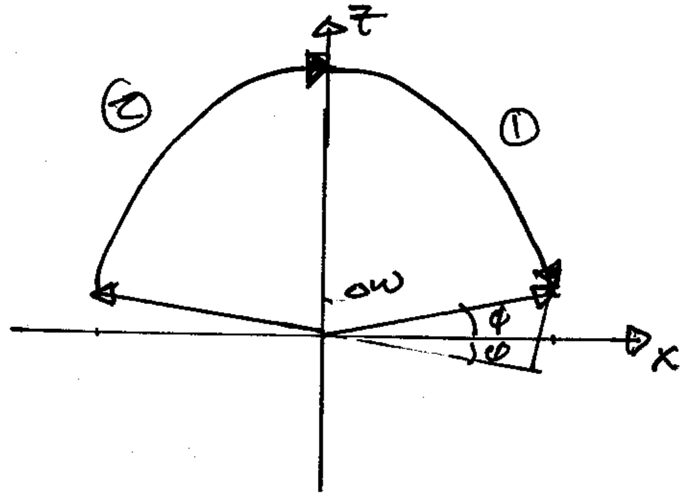
PULSE IS ADIABATIC, BECAUSE IT DEPENDS ONLY ON THE SWEEP

B_1 AMPLITUDE ONLY EFFECTS

$$\theta_{eff} = \int_{-\infty}^{0} |B_{eff}(t)|\, dt$$

WHICH CANCELS OUT AT END.

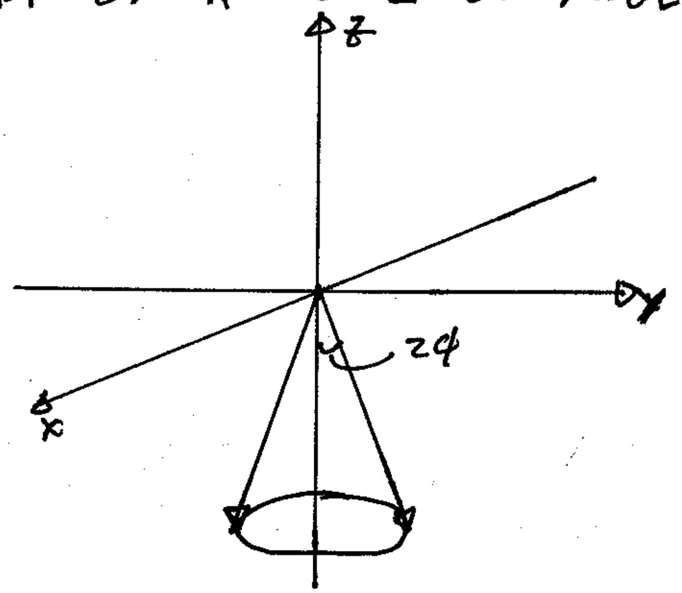
OFF-RESONANCE SENSITIVITY



$$\phi = \tan^{-1}\left(\frac{\Delta\omega}{\gamma A_0}\right)$$

AFTER SIGN CHANGE, PARALLEL COMPONENT TRACKS, PERPENDICULAR COMPONENT TRACKS AND PRECESSES.

RESULT IS INITIAL MAGNETIZATION ALONG +Z IS LEFT ON A CONE OF ANGLE $\pm 2\phi$



SIMILAR PROBLEM FOR TRANSVERSE COMPONENT.

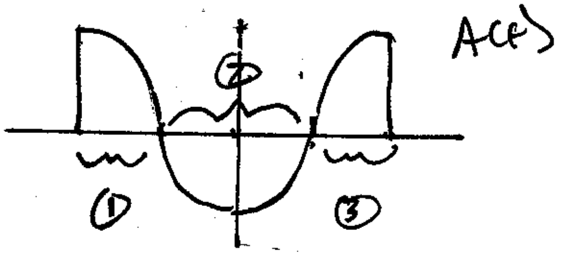
ADIABATIC ROTATION PULSES

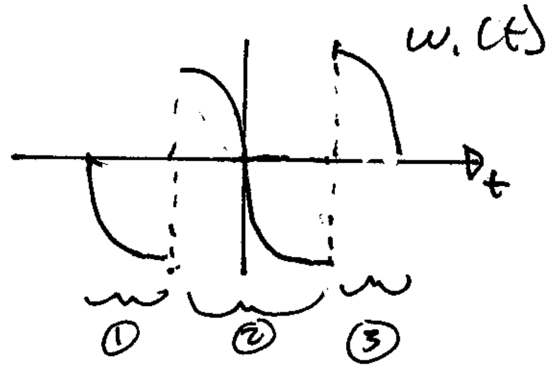
ROTATION BY SOME ARBITRARY ANGLE

- SMALL TIP ANGLE
- $\pi/2$ EXCITATION / SATURATION
- SPIN-ECHO REFOCUSING PULSE

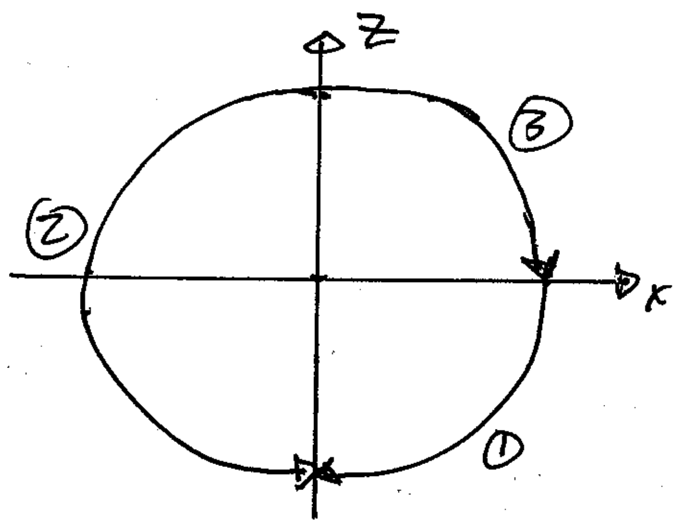
B_1 Insensitive Rotation-4 (BIR-4)

ENVELOPE / MODULATION WAVEFORMS

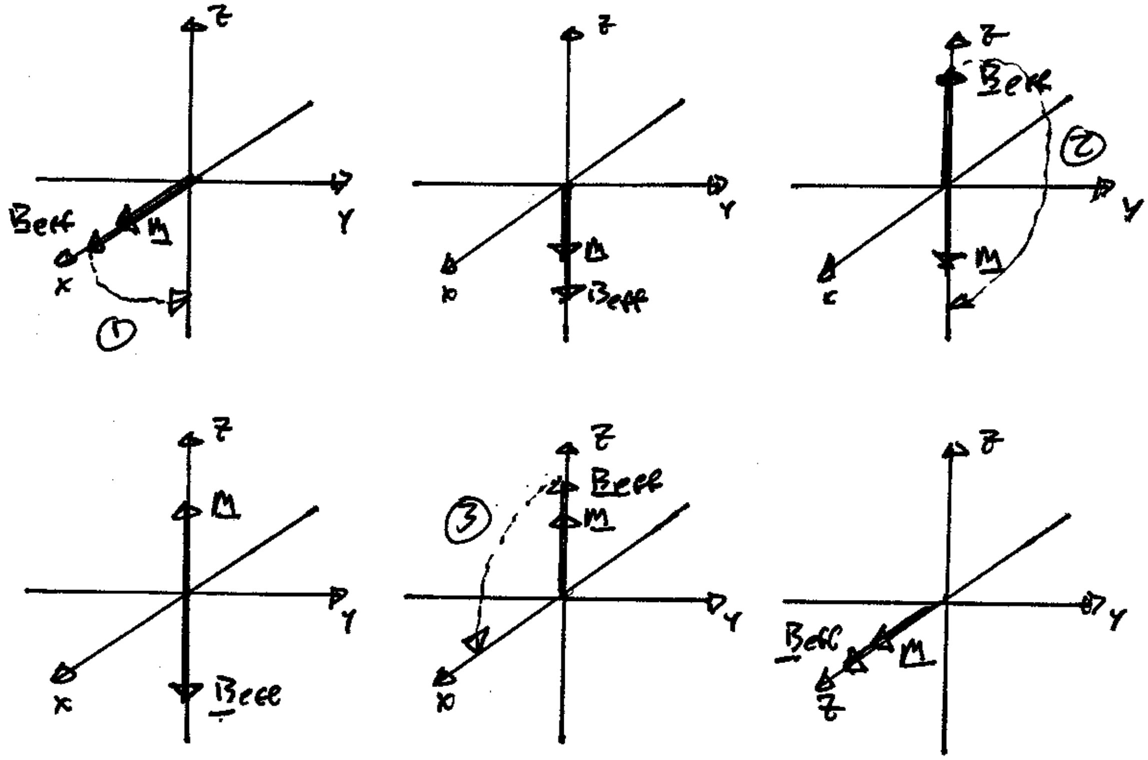




SWEEP DIAGRAM



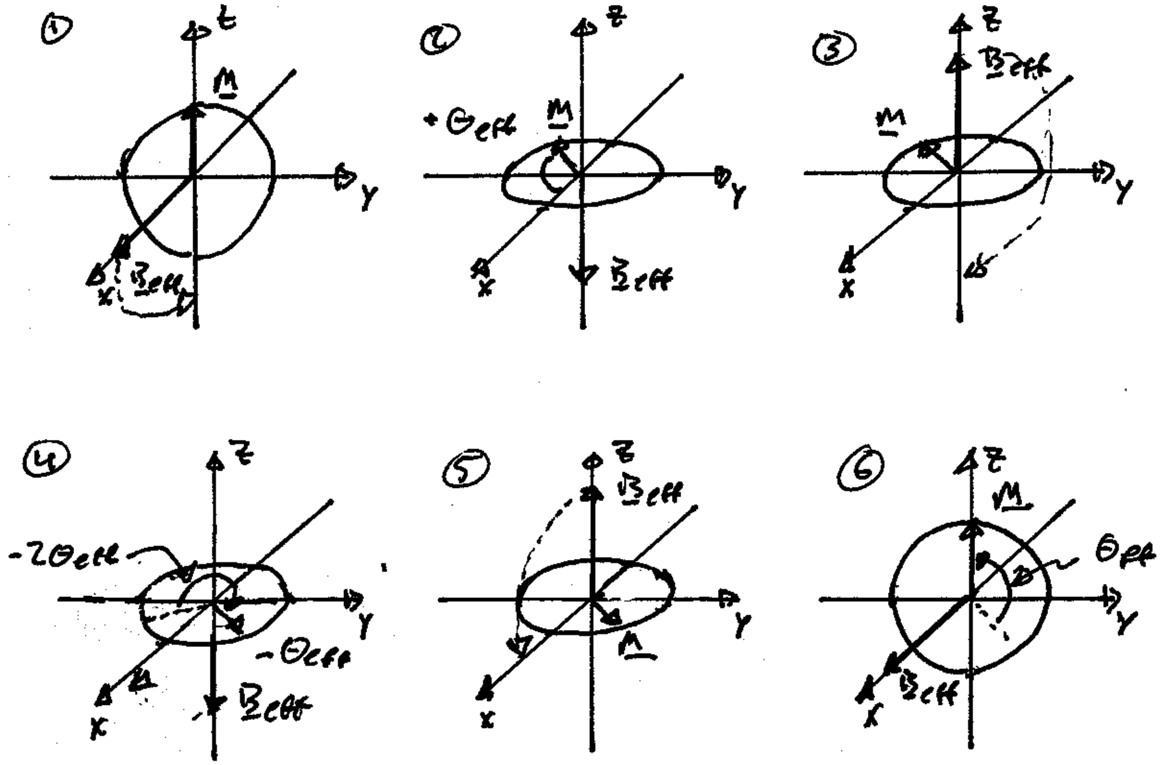
SPIN INITIALLY ALONG +X, PARALLEL



RETURNS M TO +X

DOES NOTHING!

SPIN INITIALLY PERPENDICULAR (+Z, FOR EXAMPLE)



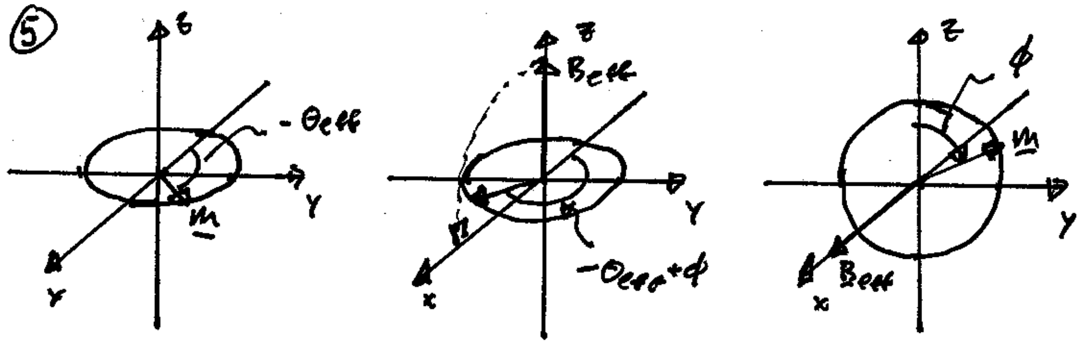
Θ_{eff} = PHASE OF HALF SWEEP

RETURNS M TO +Z

DOES NOTHING!

KEY IDEA: ADD PHASE SHIFT IN RF BEFORE FINAL SWEEP

⑤



PREPHASED BY $-\theta_{eff}$ | PHASE SHIFT BY ϕ | $-\theta_{eff}$ CANCELED LEFT WITH ϕ

ROTATION BY ϕ ABOUT THE +x AXIS!

PHASE SHIFT IS VERY ACCURATE.

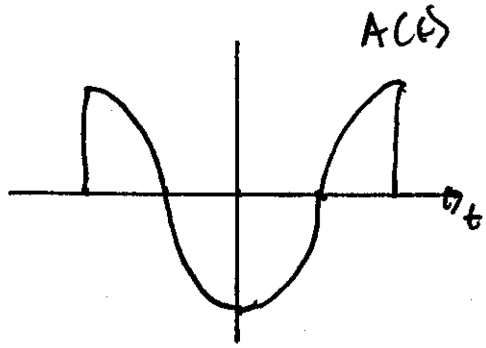
⇒ VERY ACCURATE ROTATION

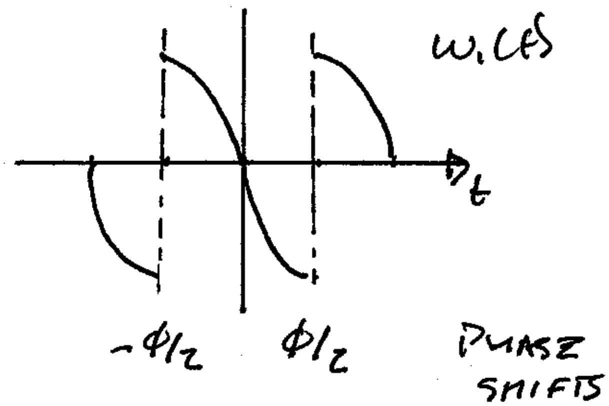
ANY ANGLE POSSIBLE
- SMALL TIP (i.e 30°)
- EXCITATION (90°)
- REFOCUSING (180°)

ALL HAVE SAME POWER

BETTER APPROACH: SPLIT PHASE SHIFT

OVER BOTH JUMPS





BIR-4